On the Impact of Phase Noise on Beamforming Performance for mmWave Massive MIMO Systems

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Content

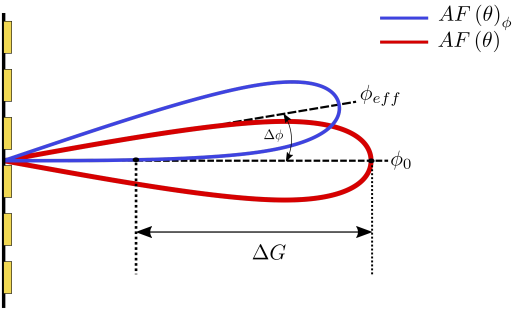
- Research context
- Contributions
- System model
- Results
- Conclusion and future work

Research context

Study of the hardware impairment effects on MU-MIMO systems :

- At the Base Station level : Beamforming Gain, Array Factor

- Focus on the Phase Noise in our study
  - Created in the RF chains : more particularly in the PLL/LO.
  - Different MIMO BS configurations : Independent or Distributed RF chains (eg. Hybrid Beamforming).



Contributions

- New analytical bounds for the asymptotic Array Factor (AF) MSE under Gaussian Phase Noise (PN) and asymptotic Gain Loss (GL) under Gaussian PN.
- Statistical study for the underlying probability distributions of the GL and effective beam steering direction.

System model

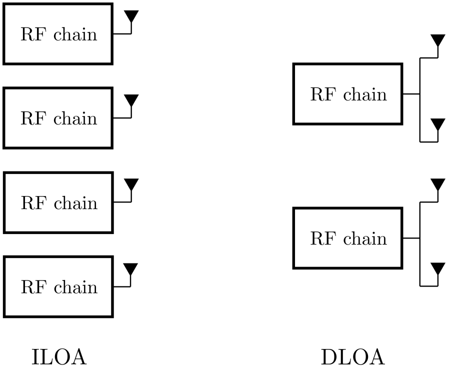


Figure: RF chains distribution scenarios

ILOA : Independent Local Oscillator Architecture
DLOA : Distributed Local Oscillator Architecture

System model

- Many PN models exist : Wiener process, Ornstein-Uhlenbeck process, Gaussian process.

- It has been shown that the Wiener process could be approximated by a Gaussian model for sub-THz bands :[1] and[2]
- Different model amongst the ILOA and DLOA scenarios

[1] Simon Bicais and Jean-Baptiste Dore. "Phase Noise Model Selection for Sub-THz Communications". In: *2019 IEEE Global Communications Conference (GLOBECOM)*. 2019, pp. 1–6.

[2] Majed Saad et al. "MIMO techniques for wireless terabits systems under Sub-THz channel with RF impairments". In: *2020 IEEE International Conference on Communications Workshops, ICC Workshops 2020 - Proceedings* (2020).

System model

- MIMO scenario with $M = \tilde{N}_b M_a$ BS antennas : $\tilde{N}_b$ blocks of M_a antennas.

- ILOA : The phase shift associated to the k^{th} antenna of the BS antenna array is defined as :

$$\phi_k \sim \mathcal{N}\left(0, \sigma_\phi^2\right), \forall k \in [\![1, M]\!] \tag{1}$$

- DLOA : The phase shift associated to the n^{th} block of antennas is defined as :

$$\phi_n \sim \mathcal{N}\left(0, \sigma_\phi^2\right), \forall n \in [\![1, \tilde{N}_b]\!] \tag{2}$$

Results

- Study of the PN impact on the AF.
- AF without PN ($\lambda/2$ inter-element spacing, ULA) :

$$AF(\theta) = \sum_{k=0}^{M-1} e^{jk\pi(\cos(\theta) - cos(\phi_0))} \tag{3}$$

- AF in the ILOA scenario :

$$AF(\theta)_\phi = \sum_{k=0}^{M-1} e^{j[k\pi(\cos(\theta) - cos(\phi_0)) + \phi_k]} \tag{4}$$

- AF in the DLOA scenario :

$$AF(\theta)_\phi = \sum_{n=0}^{\tilde{N}_b - 1} e^{j\phi_n} \sum_{k=nM_a}^{(n+1)M_a - 1} e^{jk\pi(\cos(\theta) - cos(\phi_0))} \tag{5}$$

Results

- Beamforming gain (without noise) :

$$G = \max_{\theta} \{|AF(\theta)|\} = M \quad (6)$$

- Beamforming gain (ILOA scenario) :

$$G_\phi = \max_{\theta} \left\{ \left| \sum_{k=0}^{M-1} e^{j[k\pi(\cos(\theta) - cos(\phi_0)) + \phi_k]} \right| \right\} \quad (7)$$

- Beamforming gain (DLOA scenario) :

$$G_\phi = \max_{\theta} \left\{ \left| \sum_{n=0}^{\tilde{N}_b - 1} e^{j\phi_n} \sum_{k=nM_a}^{(n+1)M_a - 1} e^{jk\pi(\cos(\theta) - cos(\phi_0))} \right| \right\} \quad (8)$$

Results

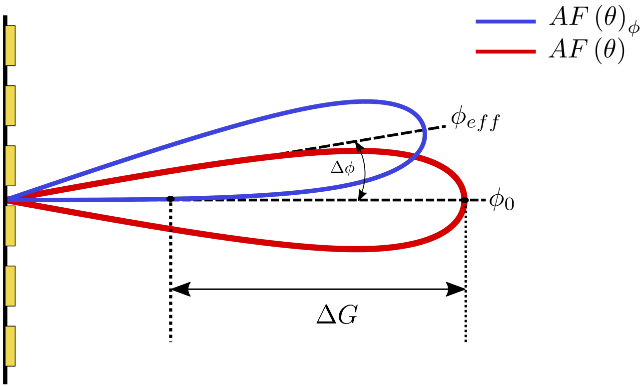


Figure: PN impact on the AF

Two main effects : Gain Loss and Beam Squint

Results

- Different metrics : AF MSE, GL, and effective beam steering direction ϕ_{eff}.
- AF MSE :

$$MSE = \frac{1}{2\pi} \int_{\theta=0}^{2\pi} \left| AF(\theta)_\phi - AF(\theta) \right|^2 \mathrm{d}\theta \tag{9}$$

- GL :

$$\Delta G = \left| \frac{AF(\phi_0)_\phi}{AF(\phi_0)} \right| \tag{10}$$

- Effective beam steering direction :

$$\phi_{eff} = \arg\max_\theta \left\{ \left| AF(\theta)_\phi \right| \right\} \tag{11}$$

$$= \arg\max_\theta \left\{ \left| \sum_{k=0}^{M-1} e^{j[k\pi(\cos(\theta) - cos(\phi_0)) + \phi_k]} \right| \right\} \tag{12}$$

Results

AF MSE bounds:

- ILOA AF MSE bound :

$$MSE \leq 2M \left(1 - \frac{1}{M} \sum_{k=0}^{M-1} \cos\left(\phi_k\right)\right) \tag{13}$$

- ILOA AF MSE asymptotic bound :

$$MSE \xrightarrow[M \to +\infty]{a.s.} 2M \left(1 - \exp\left(-\frac{\sigma_\phi^2}{2}\right)\right) \tag{14}$$

Results

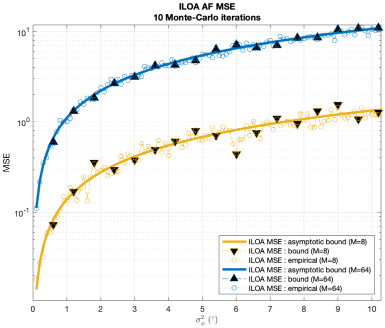


Figure: ILOA AF MSE

- Good convergence of the ILOA/DLOA AF MSE to the asymptotic bound : $M \geq 8$ for the ILOA/DLOA.

Results

PN impact on the GL :

- ILOA GL

$$\Delta G = \left| \frac{1}{M} \sum_{k=0}^{M-1} e^{j\phi_k} \right| \tag{15}$$

- DLOA GL

$$\Delta G = \left| \frac{1}{\tilde{N}_b} \sum_{n=0}^{\tilde{N}_b-1} e^{j\phi_n} \right| \tag{16}$$

- ILOA/DLOA asymptotic GL in dB :

$$\Delta G_{dB} \xrightarrow[M\to+\infty]{a.s.} \Delta G_{dB}^{as} = \frac{5\sigma_\phi^2}{\ln(10)} \tag{17}$$

Results

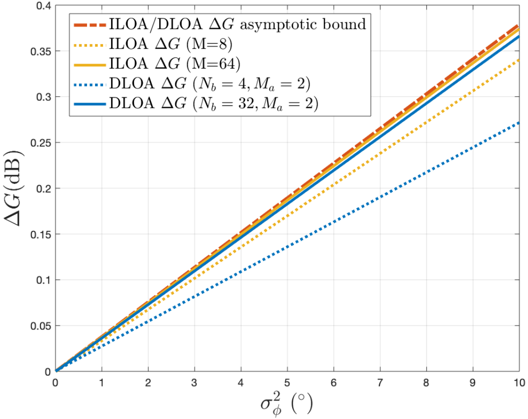


Figure: ILOA/DLOA GL

Results

- Stochastic modelling of the GL and the effective beam steering direction
- GL :

$$\Delta G = \left| \frac{AF(\phi_0)_\phi}{AF(\phi_0)} \right| \tag{18}$$

- Effective beam steering direction :

$$\phi_{eff} = \arg\max_\theta \{|AF(\theta)_\phi|\} \tag{19}$$

$$= \arg\max_\theta \left\{ \left| \sum_{k=0}^{M-1} e^{j[k\pi(\cos(\theta) - cos(\phi_0)) + \phi_k]} \right| \right\} \tag{20}$$

- Theoretical derivation of the pdfs. : Extremely hard and cumbersome because of the absolute value (not bijective) $\Rightarrow$ Use of statistical tests.

Results

- Chosen models :

$$\Delta G_{dB} \sim \Gamma\left(k_{\Delta G_{dB}}(M), \theta_{\Delta G_{dB}}(M)\right) \quad (21)$$

$$\phi_{eff} \sim \mathcal{N}\left(\mu_{\phi_{eff}}(M), \sigma^2_{\phi_{eff}}(M)\right) \quad (22)$$

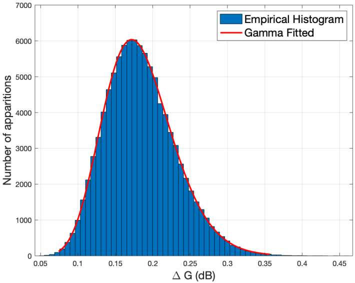


Figure: $100.000\ \Delta G_{dB}$ realisations, $(M = 32)$

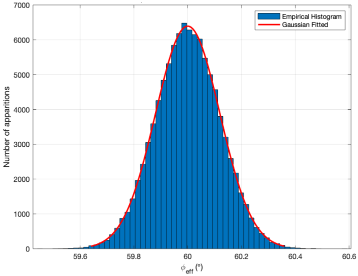


Figure: $100.000\ \phi_{eff}$ realisations, $(M = 32)$

Results

- Asymptotic behaviours of the distribution models :

$$\left(\mathbb{E}\left[\Delta G_{dB}\right]=k\theta, \mathbb{V}\left[\Delta G_{dB}\right]=k\theta^{2}\right) \underset{M\to+\infty}{\longrightarrow} \left(\Delta G_{dB}^{as}, 0\right) \tag{23}$$

and

$$\left(\mathbb{E}\left[\phi_{eff}\right]=\mu_{\phi_{eff}}, \mathbb{V}\left[\phi_{eff}\right]=\sigma_{\phi_{eff}}^{2}\right) \underset{M\to+\infty}{\longrightarrow} \left(\phi_{0}, 0\right) \tag{24}$$

- In other words, we show in simulation that the distributions asymptotically converge to Dirac distributions.
- Dirac distribution means that the metrics (effective beam steering direction and GL) become deterministic.

Results

- $M \geq 32$ is a good approximation for ϕ_{eff} distribution convergence.
- $M \geq 256$ is a good approximation for ΔG_{dB} distribution convergence

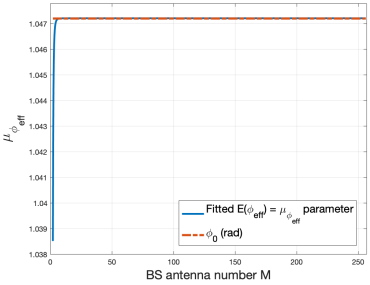


Figure: Normal distribution asymptotic convergence

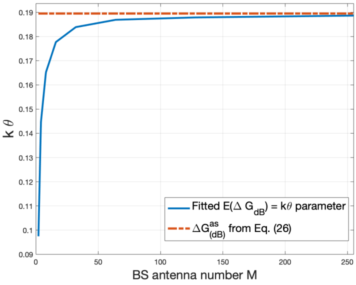


Figure: Gamma distribution asymptotic convergence

Conclusion and future work

- Physical implications :
  - Asymptotic convergence of the effective beam steering direction to ϕ_0
    - Cancellation of the beam squint : Due to an averaging phenomenon.
    - Application to MU-MIMO systems : Inter-User interferences reduction.
- Easily transposable to the DLOA and UPA scenarios.

- Contributions :
  - New analytical bounds for the asymptotic AF MSE under Gaussian PN and asymptotic GL under Gaussian PN.
  - Statistical study for the underlying probability distributions of the GL and effective beam steering direction.

- Future work : Establish analytical bounds for the generic AF definition and Gaussian PN.

Have you got any question ?

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