/ Online Unsupervised Deep Unfolding for SISO-OFDM channel estimation /

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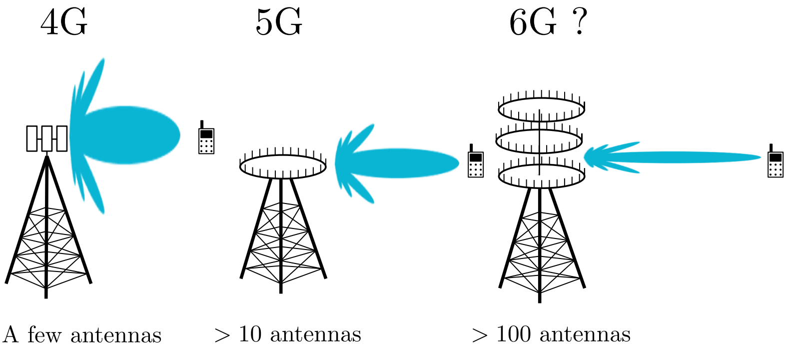
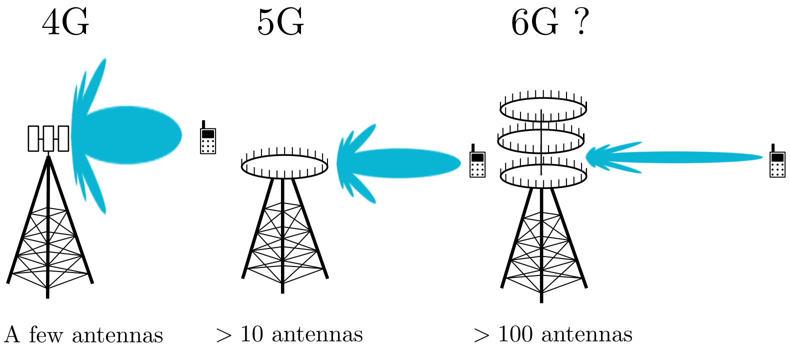
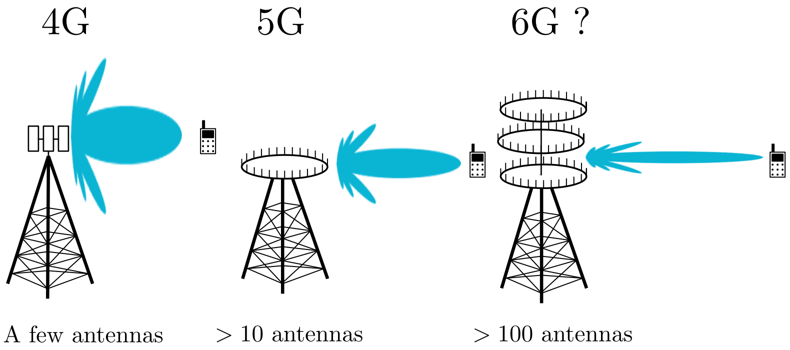

4G
5G
6G ?
A few antennas
> 10 antennas
> 100 antennas



The dimension of channels increases



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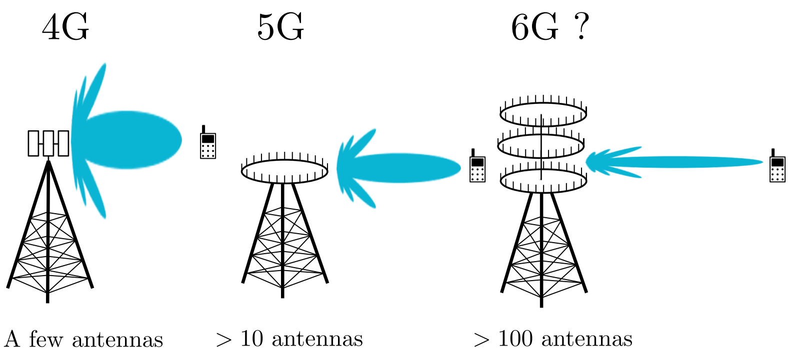

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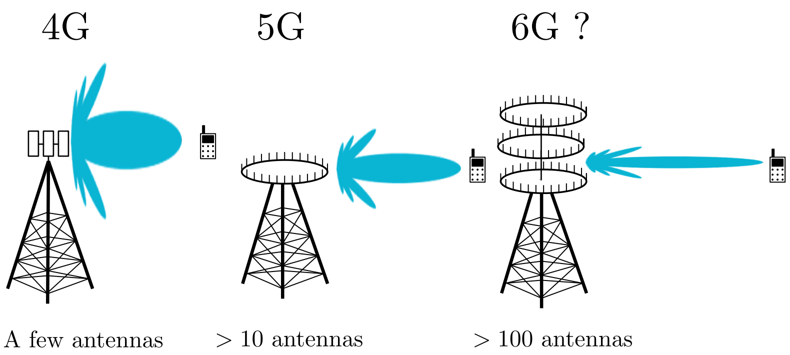

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- Consequences:
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  - We observe a ***large*** number of ***correlated*** variables, explained by a ***small*** number of ***independent*** factors.

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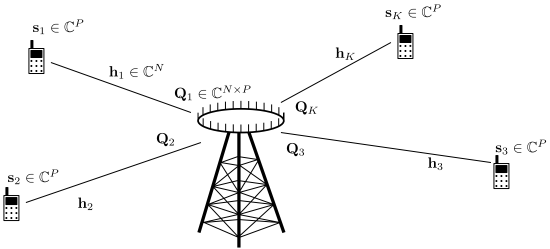
- Guide machine learning methods: reduce their complexity

Resilient channel estimation


$\mathbf{s}_1 \in \mathbb{C}^P$
$\mathbf{h}_1 \in \mathbb{C}^N$
$\mathbf{Q}_1 \in \mathbb{C}^{N \times P}$
$\mathbf{s}_K \in \mathbb{C}^P$
$\mathbf{h}_K$
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$\mathbf{Q}_2$
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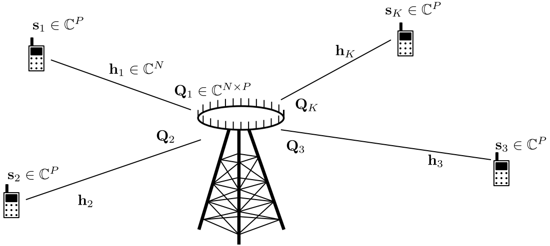
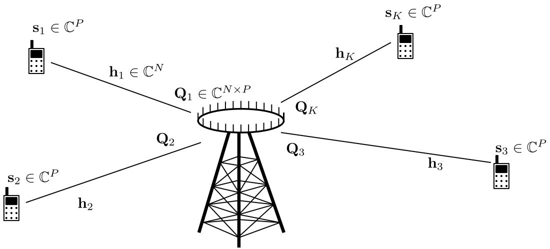


- Orthogonal pilot sequences: $\mathbf{s}_i^H \mathbf{s}_j = \delta_{i,j}$



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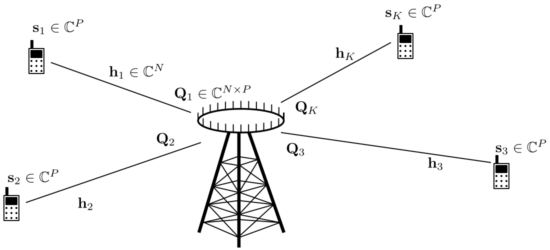
- Signal due to the k-th UE: $\mathbf{Q}_k = \mathbf{h}_k \mathbf{s}_k^T$



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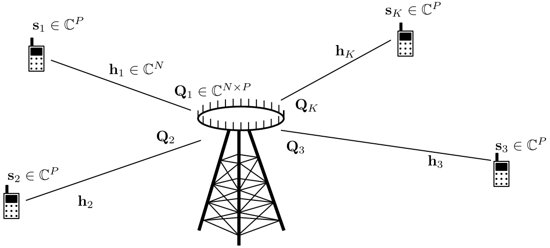

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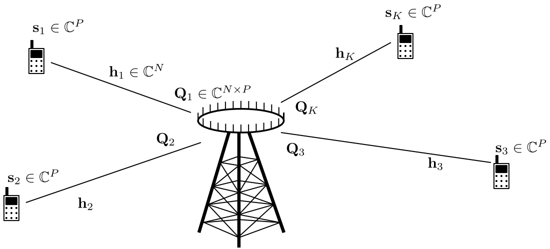

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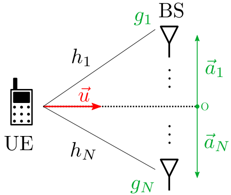
How to denoise the channels ?

SU-MIMO



Physical model:

Plane wave assumption:

$$h_O = \beta$$

$$\Downarrow$$

$$h_i = \beta g_i \mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}\vec{a}_i \cdot \vec{u}}$$

$$\Downarrow$$

$$\mathbf{h} = \sum_{l=1}^{L_p} \beta_l \begin{bmatrix} g_1 \mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}\vec{a}_1 \cdot \vec{u}_l} \\ \vdots \\ g_N \mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}\vec{a}_N \cdot \vec{u}_l} \end{bmatrix}$$

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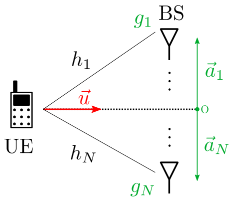

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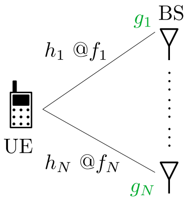
SU-SISO-OFDM



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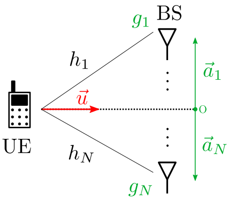

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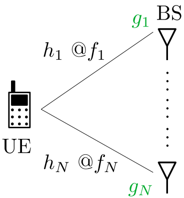

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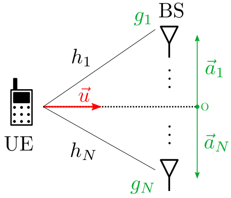

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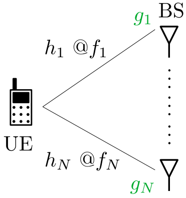

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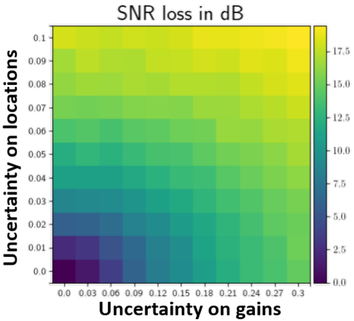

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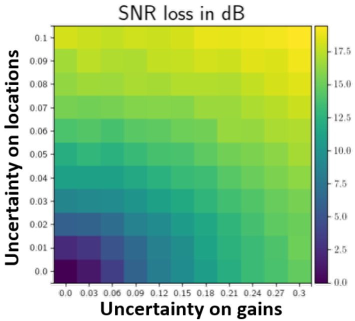

How to counter this performance loss ? Use of a neural network

- Unsupervised, online, SNR-adaptive neural network. Based on the *Deep-Unfolding*[1] approach.

[1] Balatsoukas-Stimming and Studer, “Deep Unfolding for Communications Systems: A Survey and Some New Directions”.

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- Model-based AI: MIMO channel estimation[2], SISO-OFDM (***this paper***), MIMO-ISAC[3], MIMO-OFDM-ISAC-Multi-target[4].

[1] Balatsoukas-Stimming and Studer, “Deep Unfolding for Communications Systems: A Survey and Some New Directions”.

[2] Yassine and Le Magoarou, “mpNet: variable depth unfolded neural network for massive MIMO channel estimation”.

[3] Mateos-Ramos et al., “Model-Driven End-to-End Learning for Integrated Sensing and Communication”.

[4] Mateos-Ramos et al., “Model-Driven End-to-End Learning for Multi-Target Integrated Sensing and Communication”.

Contributions

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  - Speed-up the search of the most-correlated atom

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$$\mathbf{W} = \begin{bmatrix} w_{1,1} & \cdots & w_{1,A} \\ \vdots & \ddots & \vdots \\ w_{N,1} & \cdots & w_{N,A} \end{bmatrix} \in \mathbb{C}^{N \times A} \tag{1}$$

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  - Example: $N = 256$ subcarriers and $A = 990$ atoms $\Rightarrow 506,880$ to 513 parameters

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- Non-constrained vs. constrained dictionary:

$$\mathbf{W} = \begin{bmatrix} w_{1,1} & \cdots & w_{1,A} \\ \vdots & \ddots & \vdots \\ w_{N,1} & \cdots & w_{N,A} \end{bmatrix} \in \mathbb{C}^{N\times A} \tag{1}$$

$$\overset{\diamond}{\mathbf{W}} = \begin{bmatrix} g_1 e^{-\mathrm{j}2\pi\left(f_1 - \frac{N}{2}\delta f\right)\tau_1} & \cdots & g_1 e^{-\mathrm{j}2\pi\left(f_1 - \frac{N}{2}\delta f\right)\tau_A} \\ \vdots & \ddots & \vdots \\ g_N e^{-\mathrm{j}2\pi\left(f_N + \frac{N}{2}\delta f\right)\tau_1} & \cdots & g_N e^{-\mathrm{j}2\pi\left(f_N + \frac{N}{2}\delta f\right)\tau_A} \end{bmatrix} \in \mathbb{C}^{N\times A} \tag{2}$$

  - From $2NA$ parameters to $2N+1$ parameters to learn
  - Example: $N = 256$ subcarriers and $A = 990$ atoms $\Rightarrow 506,880$ to 513 parameters

Learning parameter number is independent of the number of atoms

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  - Computationally intensive: How to speed up the process ?

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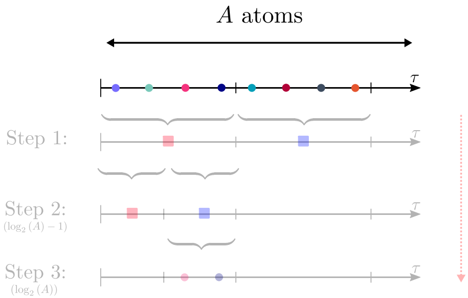

Classical approach:

- 1 step
- A correlations

Hierarchical approach:

- $\log_2(A)$ steps
- $2\log_2(A)$ correlations

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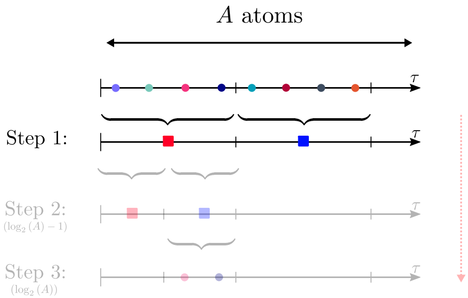

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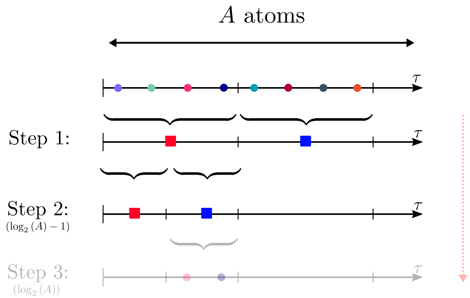

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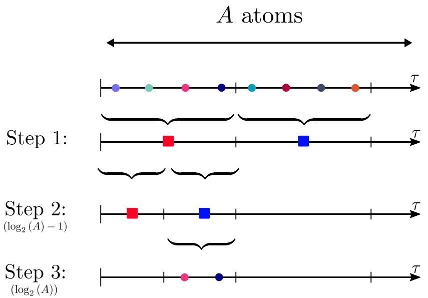

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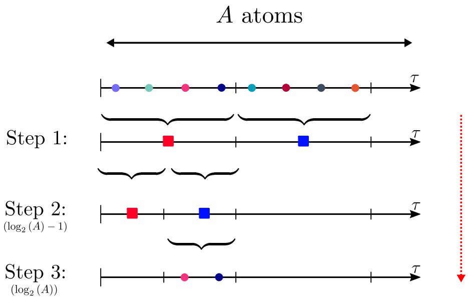

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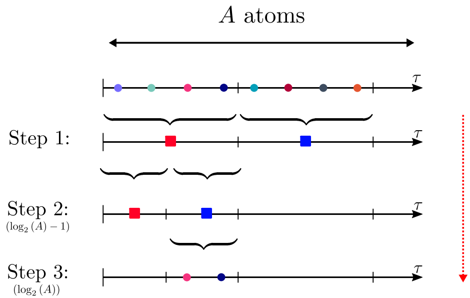

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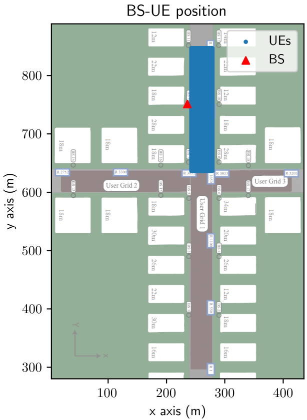
Correlation number is divided by $\frac{A}{2\log_2(A)}$

Empirical results

- DeepMIMO configuration
  - $f_0 = 3.4$GHz
  - BW = 50MHz
  - $N = 256$ subcarriers
  - Variable $\mathrm{SNR}_{\mathrm{in}}$



- DeepMIMO configuration
  - $f_0 = 3.4$GHz
  - BW = 50MHz
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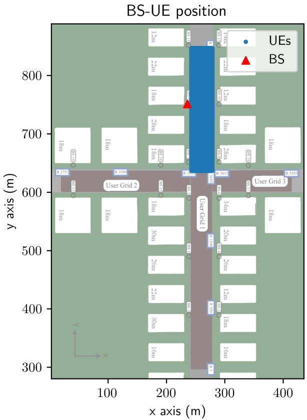
  - Variable SNR_{in}

- Imperfection models:
  - SCO: $f_i = \tilde{f}_i + i\delta f$
  - Gain imperfection: $g_i = \tilde{g}_i + n_{g_i}, \, n_{g_i} \sim \mathcal{N}\left(0, \sigma_g^2\right)$



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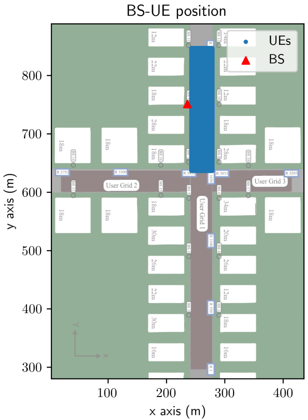
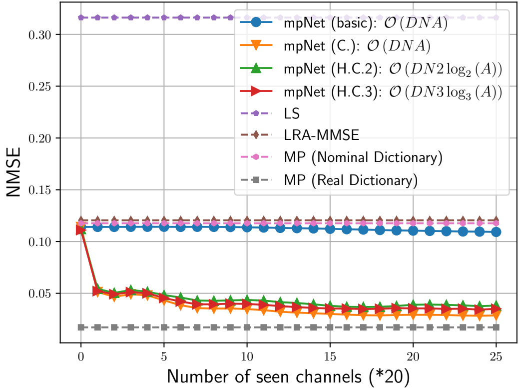
- Online (minibatch) learning
  - 10 channels per batch
  - 2000 test channels

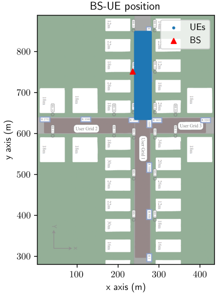


- DeepMIMO channels @3.4GHz, $N = 256$ subcarriers



SNR$_{\text{in}} = 5\,\text{dB}$, $\sigma_g^2 = 0.09$, $\xi = 40$ ppm

$$\text{NMSE} = \frac{\mathbb{E}\left[\left\|\hat{\mathbf{h}} - \mathbf{h}\right\|_2^2\right]}{\|\mathbf{h}\|_2^2}$$



- Contributions:
  - Sample complexity reduction: constrained dictionaries
  - Time complexity reduction: hierarchical search

- Link to paper: https://arxiv.org/pdf/2210.06588.pdf or QR-code:

Thank you!
Have you got any questions?

Thanks