/ Model-based learning for location-to-channel mapping /

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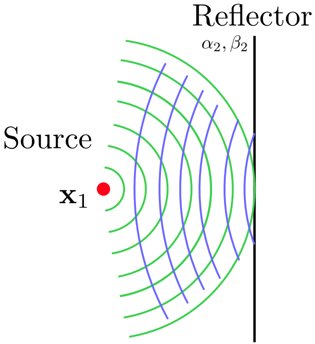

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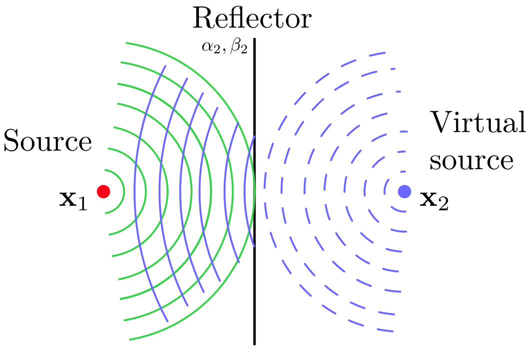

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How to efficiently learn $f_{\boldsymbol{\theta}}(\mathbf{x})$?

- Classical architecture (MLPs) are biased towards learning low frequency content[1,2]

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How to learn $f_\theta(\mathbf{x})$ without suffering from the spectral bias ?

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- Show that this model-based approach overcomes the spectral bias, and successfully learns the location-to-channel mapping

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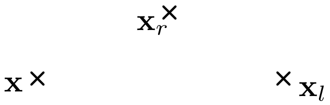

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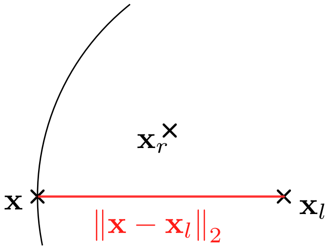

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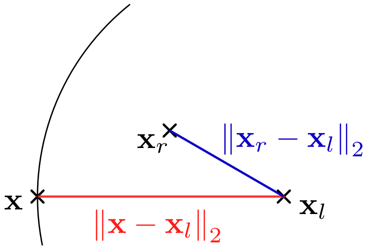

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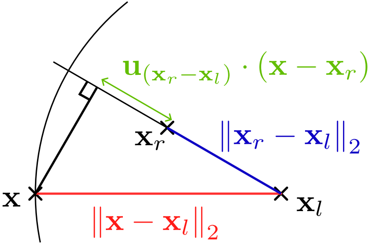

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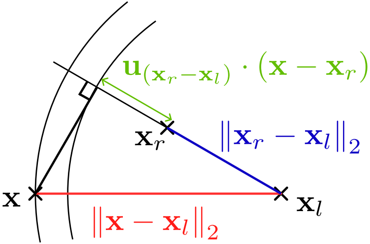

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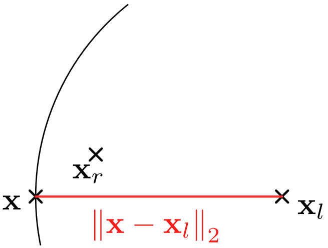

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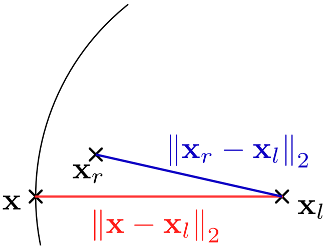

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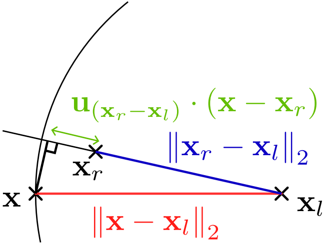

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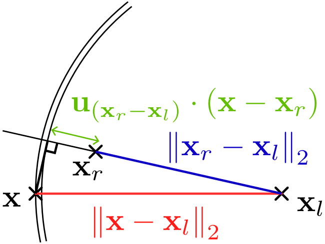

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$h(\mathrm{x})$ is locally approximated as a linear combination of planar wavefronts

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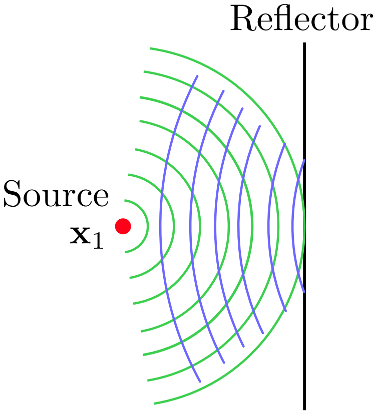

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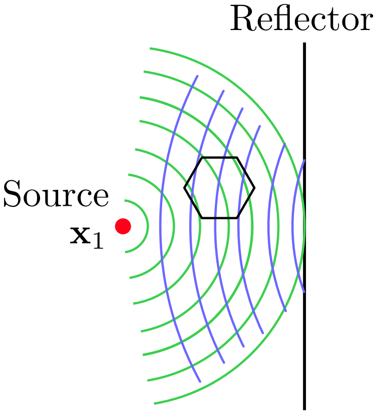

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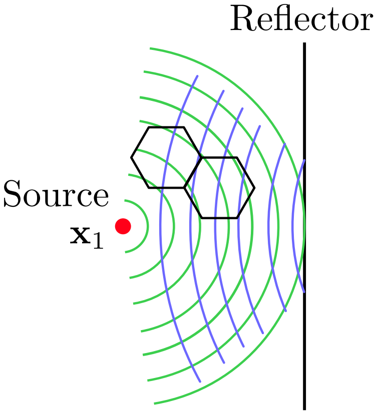

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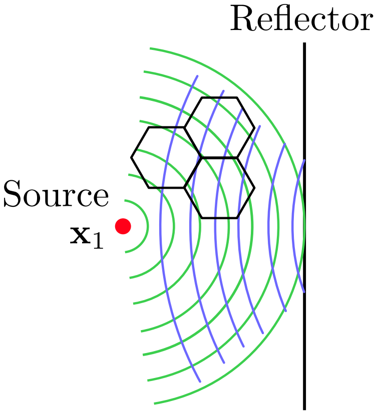

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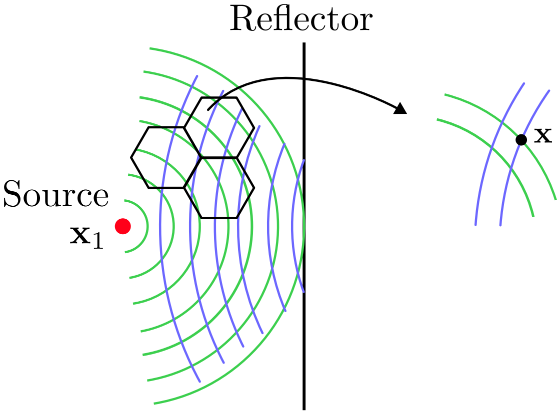

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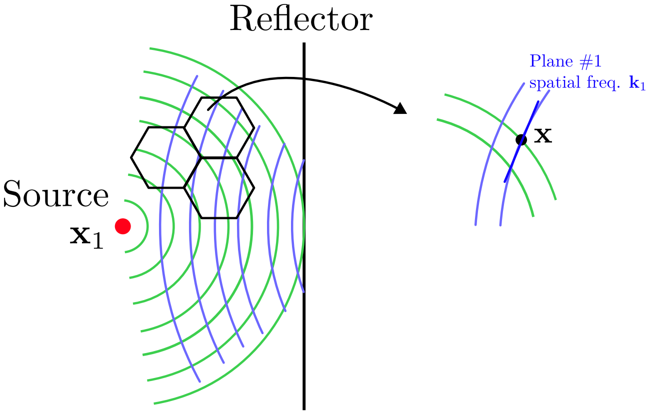

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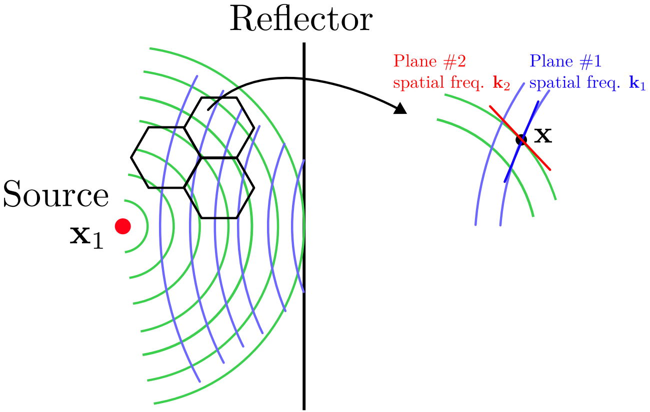

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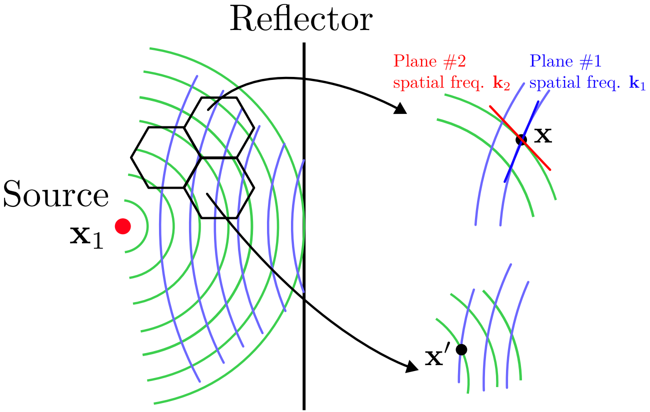

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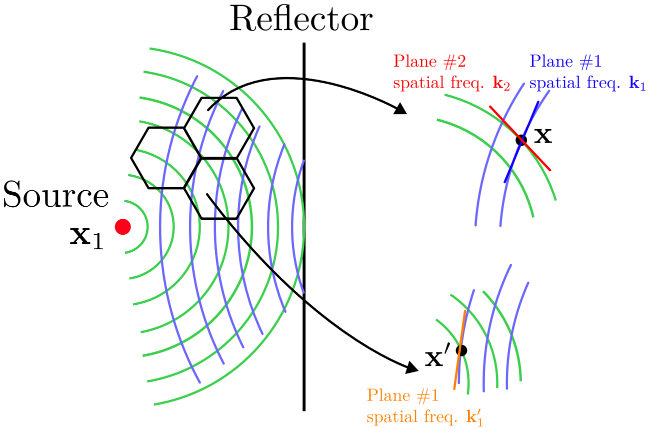

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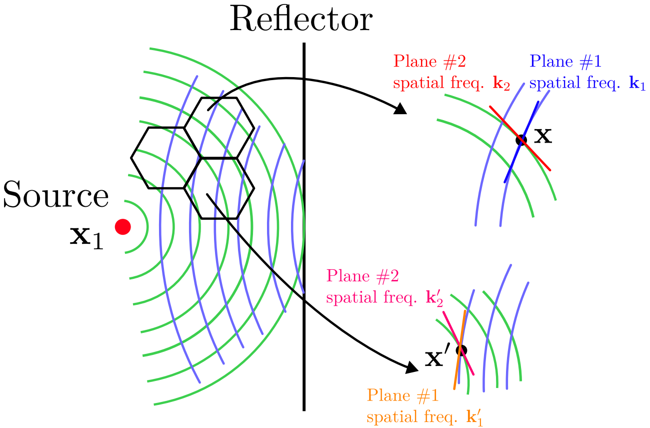

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< From local to global spatial validity >

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The local planar approximation becomes global with a well-chosen dictionary

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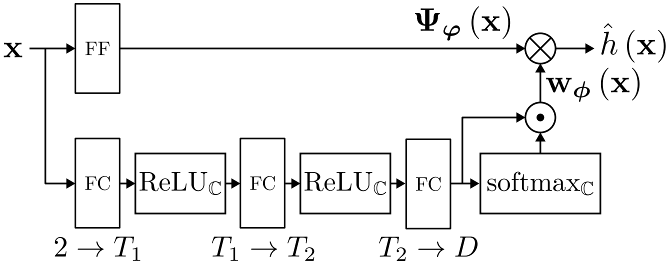

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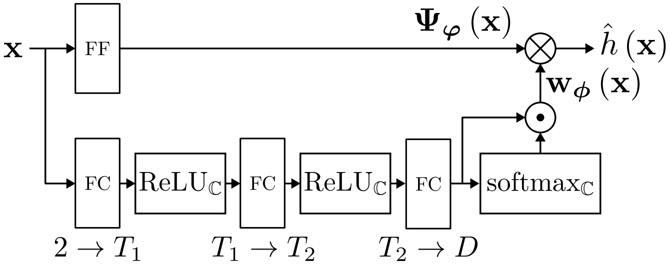

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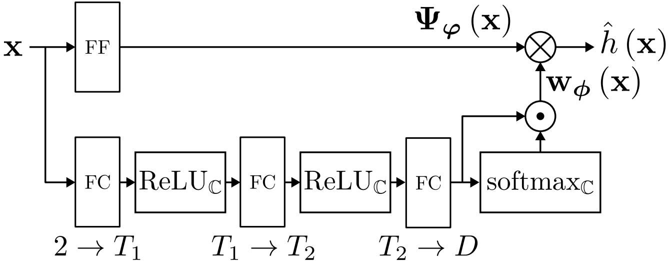
  - Compute the associated complex weights $\mathbf{w}(\mathbf{x})$, with the sparsity constraint



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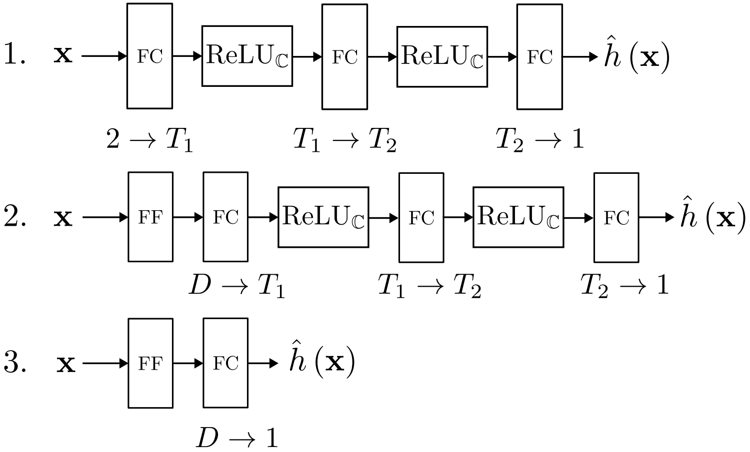
$$\text{NMSE} = 10\log_{10}\left(\frac{\left\|h\left(\mathbf{x}\right) - f_{\boldsymbol{\theta}}\left(\mathbf{x}\right)\right\|_2^2}{\left\|h\left(\mathbf{x}\right)\right\|_2^2}\right), \mathbf{x} \in \mathcal{E} \subset \mathbb{R}^2 \tag{9}$$

with $\mathcal{E}$: evaluation locations set



- 1. MLP, 2. RFF, 3. RFF lin.

< Results: scene reconstruction >

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Params.	16.8M	33.1M	4k	0.5M
NMSE$_{\text{(dB)}}$	0.16	-3.30	-3.04	$\mathbf{-20.60}$

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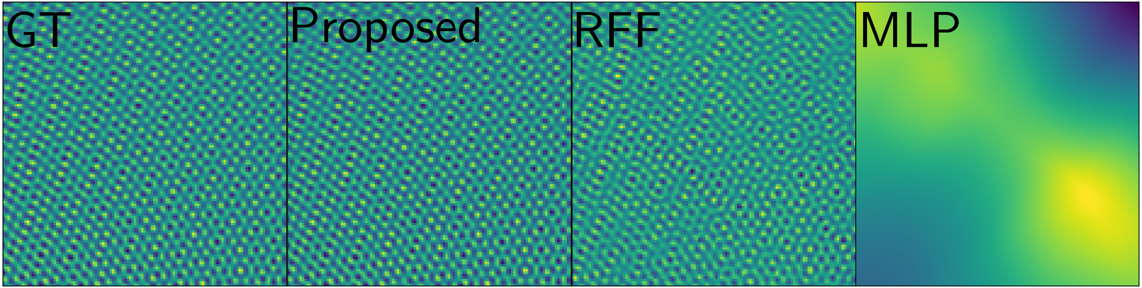

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	MLP	RFF	RFF lin.	Proposed
Params.	16.8M	33.1M	4k	0.5M
NMSE$_{\text{(dB)}}$	0.14	-2.41	-2.21	$\mathbf{-23.41}$

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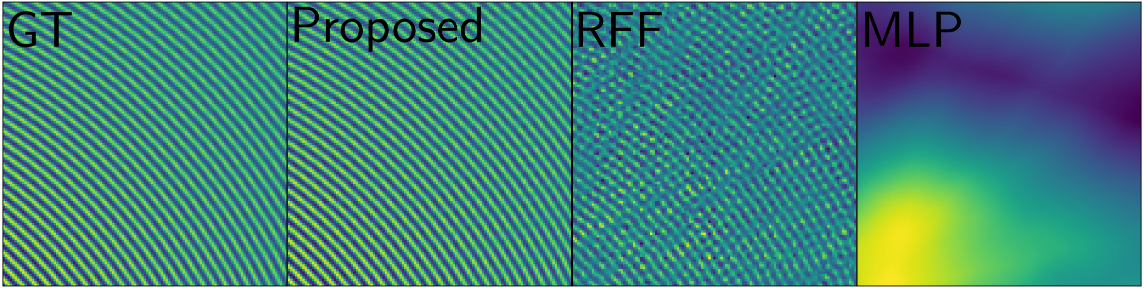

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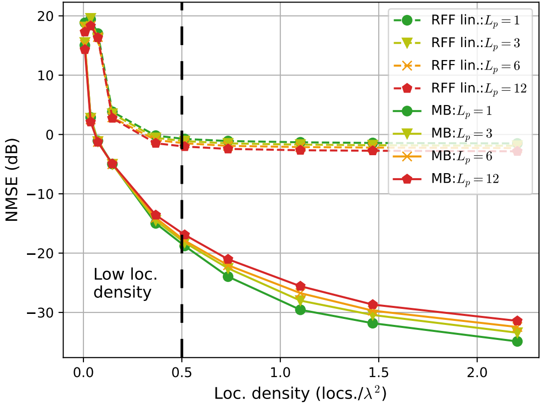

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- Link to paper: https://arxiv.org/pdf/2308.14370.pdf

Thanks